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African Journal of Mathematics and Computer Science Research

Full Length Research Paper

On the action of mobile loads on an uninterrupted cylindrical tunnel

Ismail Ibrahimovich Safarov[1], Kuldashov Nasriddin Urinovich[1] and Nuriddinov Baxtiyir Zafarovich[2]

[1]Tashkent Chemistry, Technological Institute, Tashkent, Republic of Uzbekistan.
[2]Bukhara Technological Institute of Engineering, Bukhara, Republic of Uzbekistan

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This article gives the basics of the calculation technique for the action of mobile loads of long underground transport structures such as tunnels and pipelines taking into account the influence of the earth's surface. On elastic models, the dynamic behavior of unreinforced and reinforced structures at different depths of bedding is considered, as well as the effect of the type and parameters of the running load on the stress-strain state of the rock massif. The speed of the movement of the cargo is considered subsonic, which corresponds to the modern speeds of transport in the investigated underground objects. To describe the motion of a half-space and thick-walled shells, dynamic equations of the theory of elasticity in displacement potentials are used and for thin-walled shells, the classical equations of the theory of thin shells are used. Equations are written in a moving coordinate system associated with the load. A closed system of differential equations is constructed. The system of differential equations is solved by the method of separated variables, integral Fourier transforms, the Romberg, Müller and Gauss method. From the analysis of the obtained numerical results it follows that in these cases the reinforcement of the tunnel leads to a decrease in the dynamic effect of the moving loads on the earth's surface. The earth's surface has an uneven effect on the stress-strain state of the rock massif under the action of moving loads. For a load with a shorter period, this effect is almost not noticeable and becomes noticeable for very small periods.

Key words: Thick-walled shell, stationary load, cavity, mobile coordinate system, Lame potentials.

INTRODUCTION

In many cities, plans have been made to build underground highways of considerable length as well as tunnels for new high-speed transport. Widespread development in the construction of underground main pipelines is necessary for transporting virtually the entire volume of produced natural gas, most of the oil and various cargoes. Modern transport underground structures in accordance with the requirements of reliability and durability are among the most important objects of underground construction. Along with the static calculation of such structures (Bulychev, 1989), their dynamic calculation calculation (Bakirov and Lai, 2002; Yerzhanov et al., 1989) is necessary. Among the dynamic loads and impacts on underground structures in the

*Corresponding author. E-mail: safarov54 @mail.ru

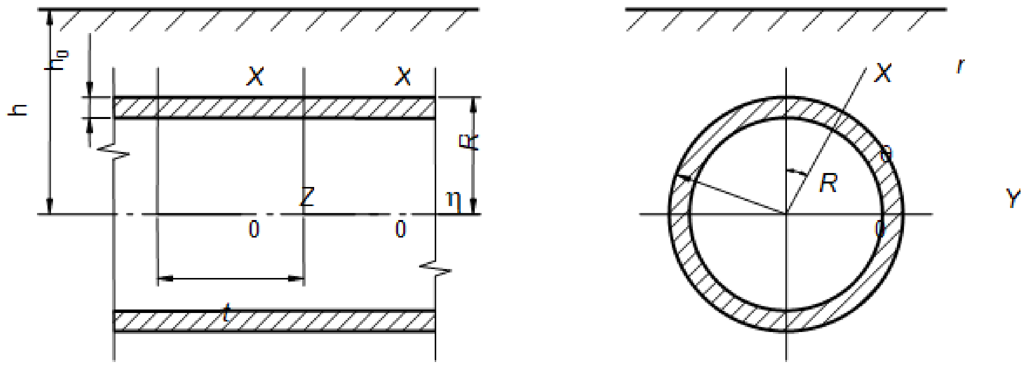

Figure 1. The calculated scheme of the reinforced tunnel and underground pipeline.

form of seismic waves of natural or artificial origin should be singled out. Difficulties in the calculation of objects in the presence of mobility of the load multiply increase in comparison with the volume of static calculations. Especially, great mathematical difficulties appear when taking into account the massiveness of the driving loads. The study of the dynamics of extended underground structures under the action of various perturbations leads to the solution of boundary value problems in the mechanics of continuous media (Jones et al., 2002; Grigolyuk and Gorshkov, 1974; Volmir, 1979; Guz et al., 2002; Hshley and Haviland, 1950).

Work in this direction with a sufficiently detailed bibliography can be found in monographs (Movchan, 1965; Carrier, 1989; Mindlin and Blech, 1953; Junger 1953). The work (Skalak and Friedman ,1958; Lugovoy et al., 1991; Hawyood, 1958; Volmir,1972; Uflyand,1968; Novatsky, 1975) and many other publications are devoted to a generalization and systematization of research results on a comprehensive study of the dynamic behavior of cylindrical shells of various designs. The stationary solution of the dynamics of an infinitely long thin cylindrical shell immersed in an acoustic medium and subjected to an axisymmetric load moving with a constant velocity in the axial direction was investigated (Pozhuyev,1984). In (Kurkchiev, 1970), the reaction of an infinitely long cylindrical shell in an acoustic medium to the action of a moving stepped plane shock wave was considered. The solution is given in generalized coordinates without taking into account the extension of the middle surface of the shell. In (Morse and Feshbakh,1960; Pozhuyev, 1977), such problems are solved by the method of integral transformations. Later, hinged-supported shells were considered in Herrmann and Baker,1967; Pozhuyev, 1978; Pozhuyev,1978; Pozhuyev, 1980). In (Slepian, 1972; Pozhuyev, 1983), the nonlinear dynamics of shells was investigated. In (Guz et al., 2002), the ax symmetric vibrations of a priestesses shell were studied under the action of a moving force, where the Bubnov-Galerkin method was applied to geometric coordinates and the Bogolyubov-Mitropolsky coordinate in time coordinate. Starting from the equation of shell motion (Bozorov et al., 1996), the dynamics of a priestess's cylinder under the action of two types of loads were studied: a concentrated normal force moving along a circle at a constant velocity and a point wise normal force moving along the axis of the cylinder.

An approximate model approach for determining vibrations on a free surface from moving loads in reinforced tunnels of a rectangular and circular profile has been applied (Pozhuyev, 1984). The action of a mobile periodic load on a circular cylindrical cavity in an elastic half-space for subsonic speeds of load motion was considered in (Watanabe,1984; Pozhuyev, 1984), where the motion of a half-space described the dynamic equations of the theory of elasticity (Yakupov,1979) in Lamé potentials (Chonan ,1981; Datta et al., 1984).

To solve problems in this paper, a model research method is used. The tunnel is modeled as an infinitely long circular cylindrical cavity located in a homogeneous and isotropic linearly elastic half-space parallel to its horizontal boundary. The cavity can be supported by a homogeneous or layered elastic shell (in which case the tunnel can be considered as an underground pipeline). The nonstationary load acts on the surface of the cavity or on the inner surface of the shell reinforcing cavity. The speed of the load is assumed to be subsonic.

STATEMENT OF THE PROBLEM FOR A CIRCULAR TUNNEL

Using the model approach for research, the tunnel was represented as an infinitely long circular cylindrical cavity with a radius $r = R$, located in a linear viscoelastic, homogeneous and isotropic half-space $x \leq h$ (Figure 1) parallel to its horizontal boundary (the earth's surface).

The half-space reaction was defined on a moving coordinate system with a constant subsonic velocity c along the cavity surface in the direction of the Z-axis of the load P.

For this, the equations of motion of an elastic medium in vector form were used (Parnes,1980; Safarov ,1992):

$$\tilde{\mu}_j \nabla^2 \bar{u} + (\tilde{\lambda}_j + \tilde{\mu}_j) grad.di\vartheta \bar{u} = \rho_j \frac{\partial^2 \bar{u}}{\partial t^2} \qquad (1)$$

Here $\vec{u}(u_x, u_y, u_z)$ is the vector of displacement of points of the medium; ρ_j is the material density; u_x, u_y, u_z is the displacement components; and ν_j is the Poisson's ratio.

$$\tilde{\lambda}_j = \frac{\nu_j \tilde{E}_j}{(1+\nu_j)(1-2\nu_j)}; \qquad \tilde{\mu}_j = \frac{\nu_j \tilde{E}_j}{2(1+\nu_j)},$$

где

where $\tilde{E}_j$ is the operational modulus of elasticity, which have the form (Safarov and Axmedov ,2018; Safarov et al.,2017)::

$$\tilde{E}_j \phi(t) = E_{0j}\left[\phi(t) - \int_0^t R_{Ej}(t-\tau)\phi(t)\,d\tau\right] \qquad (2)$$

where $\phi(t)$ is the arbitrary time function; $R_{Ej}(t-\tau)$ is the relaxation core; E_{0j} is the instantaneous modulus of elasticity; the integral terms were assumed in Equation 5 to be small, then the functions $\phi(t) = \psi(t)e^{-i\omega_R t}$, where $\psi(t)$ is a slowly varying function of time and ω_R is the real constant. Further, applying the freezing procedure (Safarov et al., 1917), we note relations (Equation 2) as approximations of the form:

$$\bar{E}\phi = E\left[1 - \Gamma^C(\omega_R) - i\Gamma^S(\omega_R)\right]\phi,$$

where $\Gamma^C(\omega_R) = \int_0^\infty R(\tau)\cos\omega_R \tau\, d\tau$ and

$\Gamma^S(\omega_R) = \int_0^\infty R(\tau)\sin\omega_R \tau\, d\tau$, respectively, are the cosine and sine Fourier images of the relaxation core of the material. As an example of a viscoelastic material, three parametric relaxation nuclei were taken as $R(t) = Ae^{-\beta t}/t^{1-\alpha}$. On the influence function, $R(t-\tau)$ is the usual requirements of inerrability, continuity (except for $t = \tau$), sign of uncertainty and monotony:

$$R > 0, \qquad \frac{dR(t)}{dt} \le 0, \;\; 0 < \int_0^\infty R(t)dt < 1.$$

where $\vec{u}$ is the vector of displacements of the environment.

Since the steady-state process is considered, the strain pattern is stationary with respect to the moving load. Therefore, it is convenient to move to a moving coordinate system $\eta = z - ct$, connected with the load P.

Then Equation 1 can be rewritten in the form:

$$\left(\frac{1}{M_p^2} - \frac{1}{M_s^2}\right)\text{grad div}\mathbf{u} + \frac{1}{M_s^2}\nabla^2\mathbf{u} = \frac{\partial^2\mathbf{u}}{\partial\eta^2}. \qquad (3)$$

Here, $M_p = c/c_p$, $M_s = c/c_s$ are the Mach numbers; $c_p = \sqrt{(\bar{\lambda} + 2\bar{\mu})/\rho}$, $c_s = \sqrt{\bar{\mu}/\rho}$ are complex propagation velocities of expansion waves: compression and shear in a medium.

TASKS OF THE ACTION OF MOBILE LOADS ON AN UNREINFORCED TUNNEL

In the theoretical aspect, the solution was based on the papers (Safarov et al., 2017; Safarov et al., 2017). In (Safarov et al., 2017), the first and second boundary-value problems of the theory of elasticity for a half-plane with a point source of stationary waves concentrated within it, the potential of which is represented in terms of cylindrical functions, are solved by the method of expanding potentials on plane waves. And in (Safarov et al., 2017), using this approach, the problem of the stationary load on the contour of a circular hole in a half-space was solved. The idea of these papers can be used on the superposition of solutions and the re-expansion of plane waves into series in cylindrical functions; in (Safarov et al., 2017), in contrast to the exact analytical solution for the subsonic case, the velocity of a moving load is less than the velocity of the Rayleigh waves.

Since the steady-state process is considered, the strain pattern is stationary with respect to the moving load. Therefore, it is convenient to move to the mobile coordinate system $\eta = z - ct$, connected with the load P.

Then Equation 1 can be rewritten in the form:

$$\left(\frac{1}{M_p^2} - \frac{1}{M_s^2}\right)\text{grad div}\mathbf{u} + \frac{1}{M_s^2}\nabla^2\mathbf{u} = \frac{\partial^2\mathbf{u}}{\partial\eta^2} \qquad (4)$$

When the load acts on the cavity surface, we have:

$$\sigma_{rj}\Big|_{r=R} = P_j(\theta, \eta), \quad j = r, \theta, \eta \tag{5}$$

where σ_{rj} is the components of the stress tensor in a medium, $P_j(\theta,\eta)$ is the components of the intensity of the mobile load $P(\theta,\eta)$.

Since the boundary of the half-space is free from loads, $x = h$:

$$\sigma_{xx} = \sigma_{xy} = \sigma_{x\eta} = 0 \tag{6}$$

Equation 1 was transformed by expressing the displacement vector of an elastic medium through Lame potentials:

$$\mathbf{u} = \mathrm{grad}\varphi_1 + \mathrm{rot}\psi \tag{7}$$

Potential ψ can be represented in the form [44]:

$$\psi = \varphi_2 \mathbf{e}_\eta + \mathrm{rot}(\varphi_3 \mathbf{e}_\eta) \tag{8}$$

where $\mathbf{e}_\eta$ «ort axis η».

With this in mind, Equation 5 takes the form:

$$\mathbf{u} = \mathrm{grad}\,\mathrm{div}\varphi_1 + \mathrm{rot}(\varphi_2 \mathbf{e}_\eta) + \mathrm{rot}\,\mathrm{rot}(\varphi_3 \mathbf{e}_\eta) \tag{9}$$

It follows from Equations 3 and 8 that the potentials φ_j satisfy the modified wave equations:

$$\nabla^2 \varphi_j = M_j^2 \frac{\partial^2 \varphi_j}{\partial \eta^2}, \; j = 1, 2, 3 \tag{10}$$

Here, $M_1 = M_p$, $M_2 = M_3 = M_s$.

The components of the stress and displacement of the material point through the potentials φ_j were expressed.

The components of the vector u (Equation 7) in cylindrical (Equation 8) and Cartesian (Equation 9) coordinate systems (Safarov et al., 2017):

$$u_r = \frac{\partial \varphi_1}{\partial r} + \frac{1}{r}\frac{\partial \varphi_2}{\partial \theta} + \frac{\partial^2 \varphi_3}{\partial \eta \partial r},$$

$$u_\theta = \frac{1}{r}\frac{\partial \varphi_1}{\partial \theta} - \frac{\partial \varphi_2}{\partial r} + \frac{1}{r}\frac{\partial^2 \varphi_3}{\partial \eta \partial \theta}, \tag{11}$$

$$u_\eta = \frac{\partial \varphi_1}{\partial \eta} + m_s^2 \frac{\partial^2 \varphi_3}{\partial \eta^2};$$

$$u_x = \frac{\partial \varphi_1}{\partial x} + \frac{\partial \varphi_2}{\partial y} + \frac{\partial^2 \varphi_3}{\partial x \partial \eta},$$

$$u_y = \frac{\partial \varphi_1}{\partial y} - \frac{\partial \varphi_2}{\partial x} + \frac{\partial^2 \varphi_3}{\partial y \partial \eta},$$

$$u_\eta = \frac{\partial \varphi_1}{\partial \eta} + m_s^2 \frac{\partial^2 \varphi_3}{\partial \eta^2},$$

where $m_s^2 = 1 - M_s^2$.

Volumetric strain:

$$\varepsilon = \mathrm{div}\,\mathbf{u} = \nabla^2 \varphi_1 \tag{12}$$

Using Hooke's law, taking into account Equations 9 and 11, we find expressions for the stress tensor components in cylindrical and Cartesian coordinates:

$$\sigma_{\eta\eta} = (2\mu + \lambda M_p^2)\frac{\partial^2 \varphi_1}{\partial \eta^2} + 2\mu m_s^2 \frac{\partial^3 \varphi_3}{\partial \eta^3},$$

$$\sigma_{\theta\theta} = \lambda M_p^2 \frac{\partial^2 \varphi_1}{\partial \eta^2} + \frac{2\mu}{r}\left(\frac{1}{r}\frac{\partial^2 \varphi_1}{\partial \theta^2} + \frac{\partial \varphi_1}{\partial r} + \frac{1}{r}\frac{\partial \varphi_2}{\partial \theta} - \frac{\partial^2 \varphi_2}{\partial r \partial \theta} + \frac{1}{r}\frac{\partial^3 \varphi_3}{\partial \theta^2 \partial \eta} + \frac{\partial^2 \varphi_3}{\partial r \partial \eta}\right),$$

$$\sigma_{rr} = \lambda M_p^2 \frac{\partial^2 \varphi_1}{\partial \eta^2} + 2\mu\left(\frac{\partial^2 \varphi_1}{\partial r^2} + \frac{1}{r}\frac{\partial^2 \varphi_2}{\partial \theta \partial r} - \frac{1}{r^2}\frac{\partial \varphi_2}{\partial \theta} + \frac{\partial^3 \varphi_3}{\partial r^2 \partial \eta}\right),$$

$$\sigma_{r\eta} = \mu\left(2\frac{\partial^2 \varphi_1}{\partial \eta \partial r} + \frac{1}{r}\frac{\partial^2 \varphi_2}{\partial \theta \partial \eta} + (1 + m_s^2)\frac{\partial^3 \varphi_3}{\partial \eta^2 \partial r}\right),$$

$$\sigma_{\eta\theta} = \mu\left(\frac{2}{r}\frac{\partial^2 \varphi_1}{\partial \theta \partial \eta} - \frac{\partial^2 \varphi_2}{\partial r \partial \eta} + \frac{(1 + m_s^2)}{r}\frac{\partial^3 \varphi_3}{\partial \theta \partial \eta^2}\right),$$

$$\sigma_{r\theta} = 2\mu\left(\frac{1}{r}\frac{\partial^2 \varphi_1}{\partial \theta \partial r} - \frac{1}{r^2}\frac{\partial \varphi_1}{\partial \theta} - \frac{\partial^2 \varphi_2}{\partial r^2} - \frac{m_s^2}{2}\frac{\partial^2 \varphi_2}{\partial \eta^2} + \frac{1}{r}\frac{\partial^3 \varphi_3}{\partial r \partial \eta \partial \theta} - \frac{1}{r^2}\frac{\partial^2 \varphi_3}{\partial \eta \partial \theta}\right);$$

$$\sigma_{\eta\eta} = (2\mu + \lambda M_p^2)\frac{\partial^2 \varphi_1}{\partial \eta^2} + 2\mu m_s^2 \frac{\partial^3 \varphi_3}{\partial \eta^3},$$

$$\sigma_{yy} = \lambda M_p^2 \frac{\partial^2 \varphi_1}{\partial \eta^2} + 2\mu\left(\frac{\partial^2 \varphi_1}{\partial y^2} - \frac{\partial^2 \varphi_2}{\partial x \partial y} + \frac{\partial^3 \varphi_3}{\partial y^2 \partial \eta}\right),$$

$$\sigma_{xx} = \lambda M_p^2 \frac{\partial^2 \varphi_1}{\partial \eta^2} + 2\mu\left(\frac{\partial^2 \varphi_1}{\partial x^2} + \frac{\partial^2 \varphi_2}{\partial x \partial y} + \frac{\partial^3 \varphi_3}{\partial x^2 \partial \eta}\right),$$

$$\sigma_{x\eta} = \mu\left(2\frac{\partial^2 \varphi_1}{\partial \eta \partial x} + \frac{\partial^2 \varphi_2}{\partial y \partial \eta} + (1 + m_s^2)\frac{\partial^3 \varphi_3}{\partial \eta^2 \partial x}\right),$$

$$\sigma_{\eta y} = \mu\left(2\frac{\partial^2 \varphi_1}{\partial y \partial \eta} - \frac{\partial^2 \varphi_2}{\partial x \partial \eta} + (1 + m_s^2)\frac{\partial^3 \varphi_3}{\partial y \partial \eta^2}\right),$$

$$\sigma_{xy} = 2\mu\left(\frac{\partial^2 \varphi_1}{\partial x \partial y} - \frac{\partial^2 \varphi_2}{\partial x^2} - \frac{m_s^2}{2}\frac{\partial^2 \varphi_2}{\partial \eta^2} + \frac{\partial^3 \varphi_3}{\partial x \partial y \partial \eta}\right).$$

Thus, to determine the components of the stress-strain state of the medium, it is necessary to solve Equation 9 together with the boundary conditions.

In cases where circular tunneling or underground pipelines are thin-walled structures, the considered model of the tunnel can be adopted as a design model, with the reinforcement of the cavity by a thin elastic cylindrical shell of thickness h_0 (Figure 1). Because of the small thickness of the shell, we assume that the surrounding array is in contact with the shell along its median surface. The load P, moving with a constant subsonic speed c in the direction of the Z-axis, acts on the inner surface of the shell.

To describe the motion of the shell, we use the classical equations of the theory of thin shells (Safarov et al., 2016):

$$\frac{\partial^2 u_{0z}}{\partial z^2}+\frac{1-\nu_0}{2R^2}\frac{\partial^2 u_{0z}}{\partial\theta^2}+\frac{1+\nu_0}{2R}\frac{\partial^2 u_{0\theta}}{\partial z\partial\theta}+\frac{\nu_0}{R}\frac{\partial u_{0r}}{\partial z}=\rho_0\frac{1-\nu_0}{2\mu_0}\frac{\partial^2 u_{0z}}{\partial t^2}+\frac{1-\nu_0}{2\mu_0 h_0}\left(P_z-q_z\right),$$
$$\frac{1+\nu_0}{2R}\frac{\partial^2 u_{0z}}{\partial z\partial\theta}+\frac{\left(1-\nu_0\right)}{2}\frac{\partial^2 u_{0\theta}}{\partial z^2}+\frac{1}{R^2}\frac{\partial^2 u_{0\theta}}{\partial\theta^2}+\frac{1}{R^2}\frac{\partial u_{0r}}{\partial\theta}=\rho_0\frac{1-\nu_0}{2\mu_0}\frac{\partial^2 u_{0\theta}}{\partial t^2}+\frac{1-\nu_0}{2\mu_0 h_0}\left(P_\theta-q_\theta\right),$$
$$\frac{\nu_0}{R}\frac{\partial u_{0z}}{\partial z}+\frac{1}{R^2}\frac{\partial u_{0\theta}}{\partial\theta}+\frac{h_0^2}{12}\nabla^2\nabla^2 u_{0r}+\frac{u_{0r}}{R^2}=-\rho_0\frac{1-\nu_0}{2\mu_0}\frac{\partial^2 u_{0r}}{\partial t^2}-\frac{1-\nu_0}{2\mu_0 h_0}\left(P_r-q_r\right) \tag{13}$$

where u_{0z}, $u_{0\theta}$, u_{0r} are the displacements of the points of the middle surface of the shell; P_z, P_θ, P_r are components of the intensity of the mobile load P; $q_z=\sigma_{rz}\big|_{r=R}$, $q_\theta=\sigma_{r\theta}\big|_{r=R}$, $q_r=\sigma_{rr}\big|_{r=R}$ are components of the reaction surrounding the shell environment; ν_0, μ_0, ρ_0 are the Poisson's ratio, the shear modulus and the density of the shell material, respectively. In the moving coordinate system, Equations 13 are rewritten in the form:

$$\left[1-\frac{\left(1-\nu_0\right)\rho_0 c^2}{2\mu_0}\right]\frac{\partial^2 u_{0\eta}}{\partial\eta^2}+\frac{1-\nu_0}{2R^2}\frac{\partial^2 u_{0\eta}}{\partial\theta^2}+\frac{1+\nu_0}{2R}\frac{\partial^2 u_{0\theta}}{\partial\eta\partial\theta}+\frac{\nu_0}{R}\frac{\partial w_{0r}}{\partial\eta}=\frac{1-\nu_0}{2\mu_0 h_0}\left(P_\eta-q_\eta\right),$$
$$\frac{1+\nu_0}{2R}\frac{\partial^2 u_{0\eta}}{\partial\eta\partial\theta}+\frac{\left(1-\nu_0\right)}{2}\left(1-\frac{\rho_0 c^2}{\mu_0}\right)\frac{\partial^2 u_{0\theta}}{\partial\eta^2}+\frac{1}{R^2}\frac{\partial^2 u_{0\theta}}{\partial\theta^2}+\frac{1}{R^2}\frac{\partial w_{0r}}{\partial\theta}=\frac{1-\nu_0}{2\mu_0 h_0}\left(P_\theta-q_\theta\right),$$
$$\frac{\nu_0}{R}\frac{\partial u_{0\eta}}{\partial\eta}+\frac{1}{R^2}\frac{\partial u_{0\theta}}{\partial\theta}+\frac{h_0^2}{12}\nabla^2\nabla^2 w_{0r}+\frac{\left(1-\nu_0\right)\rho_0 c^2}{2\mu_0}\frac{\partial^2 w_{0r}}{\partial\eta^2}+\frac{w_{0r}}{R^2}=-\frac{1-\nu_0}{2\mu_0 h_0}\left(P_r-q_r\right)$$

The motion of the half-space is described by the dynamic equations of elasticity theory in Lame potentials.

Lets consider two cases of conjugation of a shell with an environment: rigid and sliding. In these cases, the boundary conditions have the form:

(1) At sliding contact:

$$\sigma_{rj}\big|_{r=R}=0,\quad j=\eta,\theta\quad w_r\big|_{r=R}=w_{0r} \tag{14,a}$$

(2) At hard contact:

$$u_j\big|_{r=R}=u_{0j},\ j=\eta,\theta,r \tag{14,b}$$

Thus, in this formulation, in order to determine the components of displacements and stresses of the medium, it is necessary to jointly solve Equation 13, subject to the boundary conditions (Equation 14), depending on the conjugation condition of the shell with the medium.

In the moving coordinate system, we apply to the equations of motion and the boundary conditions a complex Fourier transform of the form Safarov et al., 2017):

$$\overline{\varphi}(\xi)=\int_{-\infty}^{\infty}\varphi(\eta)e^{-i\xi\eta}d\eta \tag{15}$$
$$\varphi(\eta)=-\frac{1}{2\pi}\int_{-\infty}^{\infty}\overline{\varphi}(\xi)e^{i\xi\eta}d\xi\ .$$

Writing general solutions of the transformed equations of motion of the tunnel in the form (Equations 4 to 15), we find the following system of algebraic equations for determining the dimensionless transform ants of displacements of an intermediate surface:

$$-\xi^2 U_0+i\xi G_1 w_0=-\zeta^2\frac{1-G_1}{3}G_0^2 U_0;$$
$$i\xi G_1 U_0-\frac{1-G_1}{3}G_0^2\zeta^2 w_0+\left(1+\frac{k^2\zeta^4}{12}\right)w_0-$$
$$-\frac{1-G_1}{2k}\frac{\zeta G_{11}}{\gamma}w_0=C_2 P_{10}.$$

where

$$G_1=G_2/G_1;\ k=h/a;\ P_{10}=P_0 a/Eh;$$
$$\{U_0,\ \ W_0\}=\frac{1}{h}\{U_1,\ \ W_1\};\ \ C_0^2=C\left(\frac{3}{2}\frac{\rho_1}{G_1}\right);$$

The stress at the boundary of the soft layer and elastic among (r = b) in the dimensionless form has the form:

$$\sigma_{rr}^*=\int\frac{4}{\pi}\left\{-\frac{(1-\eta)H_1^{(1)}(\overline{\alpha}a)}{\delta_1}\sin\theta+\sum^{\infty}\frac{i^{n+1}H_n(\overline{a}\ a)}{\Delta n}\sin\ n\theta\right\}e^{i\xi\eta}d\xi$$

$$\sigma_{r\theta}^*=\int\frac{2}{\pi}\left\{-\frac{i\ \overline{\beta}a^2}{\overline{\beta}^3 a^3 H_1^{(1)}(\overline{\beta}a)+8\eta\left(\frac{\overline{\beta}^2 a^2}{2}H^{(1)}{}_0(\beta a)-\overline{\beta}aH_1(\beta a)\right)}-\right.$$

$$-\frac{2}{\delta_1}\left[(1+\eta)H_1(\bar{\alpha}\,a)-\bar{\alpha}\,aH_0(\bar{\alpha}\,a)cos\theta\right]-$$

$$\left.-2\sum_{n=2}^{\infty}\frac{i^{n+1}\left[-nH_n^{(1)}(\bar{\alpha}\,a)+(\bar{\alpha}\,a)H_{n-11}(\bar{\alpha}\,a)\right]}{\Delta n}\cos n\theta\right\}e^{i\xi\eta}$$

$$где\quad \delta_1=-4\eta H_1^{(1)}(\bar{\alpha}\,a)H_1(\bar{\beta}\,a)+(1+\eta)\bar{\alpha}\,aH_0(\bar{\alpha}\,a)+H_0(\bar{\beta}\,a),$$

$$\Delta n=n\bar{\alpha}\,aH_{n-1}(\bar{\alpha}\,a)H_{n1}(\bar{\beta}\,a)+n(\bar{\beta}\,a)H_{n-1}(\bar{\beta}\,a)H_n(\bar{\alpha}\,a)-$$
$$-\bar{\alpha}\bar{\beta}a^2H_{n-1}^{(1)}(\bar{\alpha}\,a)H_{n-1}^{(1)}(\bar{\beta}\,a)$$

Here, $\delta = c/c_в$ is the ratio of the density of the environment to the density of the soft layer; $\bar{\alpha},\bar{\beta}$ are functions of ξ and η.

The following expression for the load was found which is transferred to the shell from the side of the soft layer:

$$\bar{q}_{rc}=-G_1\frac{\xi}{q}C_1w_0-C_2P_0(\xi);$$

$$C_1=\sum_{j=1}^{4}\frac{A_{4j}\big|_{k_{c1}=0}B_{3j}}{\det\|A_{ke}\|};$$

$$C_2=\sum_{j=1}^{4}\frac{(-1)^jA_{4i}\big|_{k_{c1}=0}B_{3j}}{\det\|A_{ke}\|}.$$

Elements of the determinant $\det\|A_{ke}\|$ is computed then formula $A_{11}=-2M_1;\ A_{12}=-a_{11};\quad A_{13}=nM_{12};$
$A_{14}=-A_{13};\qquad A_{21}=-S_1A_{11};$
$A_{22}=A_{12}*k_0(z_1)/k_1(z_1);$
$A_{23}=A_{13}*I_0(z_2)/I_2(z_2);\ A_{24}=A_{13}*k_0(z_1)/k_1(z_2);$
$A_{31}=\frac{1}{2}A_{11};\ A_{32}=-\frac{1}{2}A_{11};$
$A_{41}=n_1k_0(z_3)/k_1(z_4)-2A_{21}/(z_3/M_2);$
$A_{31}=A_{13}/n_1;\quad A_{34}=-A_{13}/n_1;$
$A_{42}=n_1I_0(z_3)/I_1(z_4)-2M_1S_1(z_3)/I_1(z_4);$

$A_{43}=-2M_{12}^2\left(k_0(z_5)/k_1(z_6)+I_1(z_5)/I_1(z_6)/(z_6/M_2)\right);$

$A_{44}=-2M_{12}^2\left(I_0(z_5)/k_1(z_6)+I_1(z_6)/k_1(z_6)/(z_6/M_2)\right);$

where is $m_1=\sqrt{1-M_p^2};\ m_{12}=\sqrt{1-M_S^2};\ z_1=M_1\eta;$
$z_2=M_{121}\eta;\ z_3=M_1\eta;\ z_4=m_1\eta(1+k_{11});\ z_5=m_1\eta;$
$k_{11}=(b-a)/a;\quad k_{1_0}\ \ k_1$ is the modified Neumann functions; $I_{1_0}\ \ I_1$ is the modified Bessel functions. The general solution of the equations of the motion of the environment has the form $(C_f<C_S<C_p)$:

$$\varphi(r,\xi)=A_n(\xi)k_n(m_1\xi r)+B_n(\xi)I_4(m_1\xi r)$$
$$\psi(r,\xi)=C_n(\xi)k_n(m_{i2}\xi r)+D_n(\xi)I_4(m_{12}\xi r)\qquad(16)$$

His expression for the original transform of the normal displacement has the form:

$$w_0=-\frac{1-\nu}{m}\sum_{i=o}^{\infty}\left\{\int_{-\infty}^{\infty}\frac{\Delta_1\left[a\cos(\zeta\eta)-\zeta\lim(\zeta\eta)\right]d\xi}{\left[a^2+\zeta^2\right]\det|A_{ke}|}\right\}\qquad(17)$$

Defining $\Delta_j\quad(j=1.2.\ .\ .5)$ is obtained from $\det\|A_{ke}\|$ by replacing j = 20 by the column C with the elements {0, 0, 1, 0, 0}. After this function: $A(\zeta).....D(\zeta)$ from Equation 16 can be calculated from formulas:

$$\{A,B,C,D\}=\frac{a^2}{\xi^2\det\|A_{ke}\|}\left\{\frac{A_1^1}{k_1(m_1\xi)};\quad-\frac{A_2^1}{I_1(m_2\xi)};\quad-i\frac{aA_3^1}{\zeta k_1(m_{12}\xi)};\ i\frac{aA_4^1}{\zeta I_1(m_{12}\xi)}\right\}$$

$$A_j^1=\frac{\xi}{a}M_{3k}w_0+P_0|m_{4k}|G_1\ (\kappa\ =\ 1,......4)$$

where $m_{ie}-$ minors of the element A_{je} .

For a specific value of the load velocity C, the denominators under the integral expressions in formulas (Equation 14) are transcendental functions with respect to ζ C real coefficients depending on C, as well as on the mechanical parameters of the shell and the layer. Analysis of the integrals of treatment must begin with consideration of cases (Safarov,1992) $D(\xi,C_0)=0,$ which is equivalent to the construction of the dispersion relation in the corresponding problem of propagation of free waves and the determination of the denominator from the dispersion curves of the roots for the chosen velocity of the load C at C< C_5 are possible for cases. Figure 2 shows the change in the movement of the filler, depending on the thickness of the bodies for different values of the rigidity of the aggregate. As can be seen from the drawing (γ = 100, 50,10,2), that for a sufficiently rigid layer (γ = 100), the deflections of the shell essentially

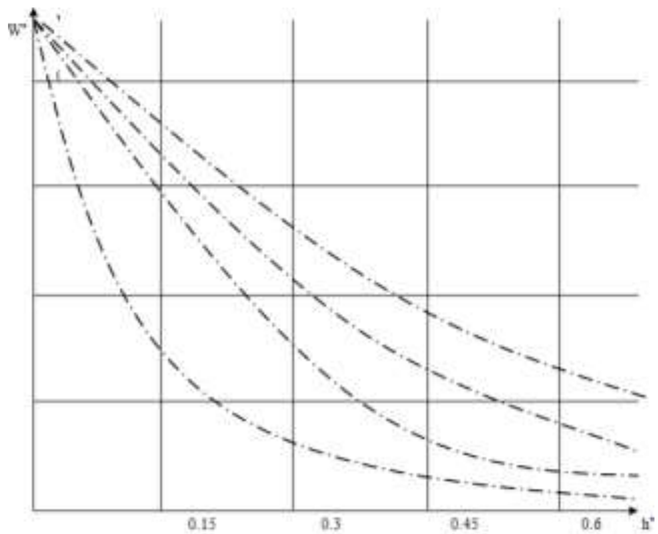

Figure 2. Shell deflections as a function of thickness.

decrease Safarov et al., 2017; Safarov et al., 2017).

(1) For a given speed C, there are one or two different denominator roots.
(2) For some values of C, the denominator has a double root. This case corresponds to a minimum of the corresponding dispersion curve in Figure 2. Such a velocity is called resonance and is denoted by C^x. A resonance effect appears, or which deflections and contact pressures tend to infinity.
(3) For a given value of C, the denominator has no roots on the real axis, as shown in Figure 2, this will be either, $C<C_\Phi$ (up to resonance mode). At this speed of motion, the inversion integrals are not special and can be found by effective numerical methods.

Dividing the integral (Equation 17) into two terms:

$$w_0 = \frac{1}{\pi}\int_0^\infty x_1(\Omega)d\Omega \quad \text{And}$$

$$w_0 = \frac{1}{\pi}\int_{\omega_1}^{\omega_2} x_1(\Omega)d\Omega \qquad (18)$$

The value of the integral (Equation 18) was found by the numerical method [50]. When the integral is calculated by the Romberg method, it is necessary to repeatedly calculate the integrand function. The inverse Fourier transform (Equation 29) was numerically fulfilled. It is shown that at an integration step of 1.01, the error of the procedure does not exceed 0.3 to 0.5%.

CONFLICT OF INTERESTS

The authors have not declared any conflict of interests

CONCLUSIONS

(1) From the analysis of these results, it follows that for any conjugation of the shell with the array, the reinforcement of the tunnel leads to a decrease in radial displacements and compressive axial stresses ($\sigma_{\eta\eta}$). The effect of the shell on the nature of the change in normal stresses ($\sigma_{\theta\theta}$) is somewhat different; these stresses increase in the central parts of the tunnel arch. As the thickness and stiffness of the sheath material increase, the displacement and stresses decrease. Contact conditions also affect the stresses and permeations of the contour of the section.
(2) All the considered load velocities, with a relatively small period T = π / 4 and the fluidity of the medium (0 <A <0.48), the components of the stress-strain state of the earth's surface are practically zero. With a decrease in the period (T / h <0.4), as calculations have shown, an entire region of the array with zero components begins to form from the earth's surface, which covers the entire array with a sufficiently small period, except for a small thickness of the layer around the tunnel.

CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

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Full Length Research Paper

A study on exact solution of the telegraph equation by (G'/G)-expansion method

Muhammad Minarul Islam* and Mohammad Sanjeed Hasan

Department of Mathematics, Bangabandhu Sheikh Mujibur Rahman Science and Technology University, Gopalganj, Bangladesh.

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In this article, an exact solution of the Telegraph equation is solved by (G'/G)-Expansion method and this is one of the most popular example both linear and non-linear partial differential equations. The (G'/G)-Expansion method is simple and powerful analyzed for getting some sets of exact solutions. To develop the theory and to visualize the graph, the mathematical software MAPLE was used. This method also gives us various kind of heat and wave equations which are implemented not only various types of heat and wave equation but also take a good decision from figure.

Key words: (G'/G)-Expansion method, Telegraph equation, periodic heat, periodic wave.

INTRODUCTION

Nonlinear partial differential equations play a very important role not only in engineering sciences such as quantum mechanics, fluid mechanics but also in mathematical and chemical physics, for example geochemistry, optical fibers, plasma physics, meteorology and biology. As non-linear partial differential equations are difficult to solve, so many powerful methods are applied to solve such as Hirota's bilinear transformation method (Hirota, 1973), the tanh-function method (Malfliet, 1992), the extended tanh-coth method (Nassar et al., 2011), the exp-function method (Islam et al., 2015), the adomian decomposition method (Adomian, 1994), the function-Expansion method (Zhou et al., 2003), the auxiliary equation method (Sirendaoreji, 2011), the Jacobi elliptic function method (Ali, 2011), the modified exp-function method (He et al., 2012), the (G'/G)-Expansion method (Zaman and Sayeda, 2013; Manafianheris, 2012; Taghizade and Neirameh, 2010; Taha and Noorani, 2014; Naher and Abdullah, 2012; Verma et al., 2013), the homotopy perturbation method (Mohiud-Din, 2007), the homogeneous balance method (Zayed et al., 2004), the modified simple equation method (Khan and Akbar, 2013), He's polynomial (Mohyud-Din et al., 2009), asymptotic method and nano-mechanics (He, 2008), vibrational iteration method (Mohyud-Din et al., 2010), the casoration formulation (Ma and You, 2004), the frobenius integrable decomposition (Ma and You, 2004), the extended multiple Riccati equations expansion method (Ma et al., 2007), and the enhanced (G'/G)-Expansion method (Islam et al., 2013a, b). Various types of nonlinear equations have been solved by many researchers by different methods.

*Corresponding author. Email: minarul_math@yahoo.com.

The Zakharov-Kuznetsov equation and the (2+1) dimensional Burgers equation, which graphical behaviors have been shown by Islam et al (2013) by enhanced (G'/G)-Expansion method where the Boussinesq equations (Islam et al., 2013b) have been solved by Akber and Kamruzzaman with this method. Generalized Reccati equation (Zaman and Sayeda, 2013), BBM and MBBM equation (Manafianheris, 2012), TRLW and Gardner equations (Taghizade and Neirameh, 2010), Fisher's equation (Taha and Noorani, 2013), the Fourth Order Boussinesq Equation (Naher and Abdullah, 2012), shallow water wave equation (Verma et al., 2013) are (G'/G)-Expansion method. To the best of the authors' knowledge, Telegraph equation has not been solved by using (G'/G)-Expansion method. Here, Telegraph equations are solved by using (G'/G)-Expansion method which has been already proposed by the Chinese mathematicians (Wang et al., 2008) for which the wave and heat solution of the non-linear evolution equations are obtained.

THE (G'/G)-EXPANSION METHOD

The (G'/G)-Expansion method for finding of non-linear evolution equation (Hirota, 1973) was discussed. Suppose that a non-linear equation say in two independent variables x and t is given by

$$P(u,u_x,u_t,u_{xx},u_{tt},u_{xt},\ldots\ldots\ldots)=0 \tag{1}$$

where $u=u(x,t)$ is an unknown function, P is a polynomial in $u=u(x,t)$ and its partial derivatives in which the highest order derivatives and the nonlinear terms are involved. The following shows the main steps of improved (G'/G)-Expansion method.

Step 1. Suppose that $u=u(x,t)$. The variable allows the reducition to an ordinary differential equation for $u=u(\xi)$

$$P(u,u',u'',\ldots\ldots\ldots)=0 \tag{2}$$

where prime denotes the derivative with respect to ξ.

Step 2. Suppose the solution of equation can be expressed by a polynomial in (G'/G) as follows:

$$u(\xi)=\sum_{i=0}^{N}c_i\left(\frac{G'(\xi)}{G(\xi)}\right)^i \tag{3}$$

where c_i are real constants with $c_i\neq 0$ to be determined. N is a positive integer to be determined. Here, N is determined by considering the homogeneous balance between the highest order derivatives and non-linear terms appearing in the equation.

If homogeneous balance is not possible, then integrating the Equation 2 and the constant term of integration supposed to be zero and then we determined the value of N by above procedure.

The function $G(\xi)$ is the solution of the auxiliary linear ordinary differential equation:

$$G''(\xi)+\lambda G(\xi)+\mu G(\xi)=0 \tag{4}$$

where λ and μ are real constants to be determined.

Step 3. Substituting Equation 3 into Equation 2 and using second order linear ordinary differential Equation 4. Separate all terms with same order of (G'/G) together, the left hand side of Equation 2 is converted into another polynomial in (G'/G). Equating each coefficient of polynomial to zero. Then we get algebraic equation for c_i, λ and μ.

Step 4. Since the following general solution of Equation 4 has been well known, then substituting c_i and general solution of Equation 4 into Equation 3. We have more solution of non-linear partial differential Equation 1.

TELEGRAPH EQUATION

The Telegraph equation is

$$u_{xx}=au_{tt}+bu_t+cu \tag{5}$$

where u is a function of x and t.

According to method described earlier, we make the transformation $u(x,t)=u(\xi),\xi=x-dt$. Then we get

$$\left(1-ad^2\right)u''+bdu'-cu=0$$

where prime denote the derivative with respect to ξ.

Now by integrating this equation and let the integrating constant to be zero, we get

$$\left(1-ad^2\right)u'+bdu-\frac{1}{2}cu^2=0 \tag{6}$$

Now balancing u' and u^2, we get $N=1$.

Therefore, we can write the solution of equation in form

$$u(\xi)=\alpha_0+\alpha_1\left(\frac{G'}{G}\right) \tag{7}$$

where $\alpha_1\neq 0$ and $G=G(\xi)$. Now from Equations 4 and 7, we derive

$$u'(\xi) = -\alpha_1 \left(\frac{G'}{G}\right)^2 - \alpha_1 \lambda \left(\frac{G'}{G}\right) - \alpha_1 \mu \quad (8)$$

Substituting Equation 8 to Equation 7 into Equation 6, setting the coefficient $\left(\frac{G'}{G}\right)^i, i = 0,1,2$ to zero, we obtain a system of algebraic equations for $a_0, a_1, a_2, c, \lambda, \mu$ as follows:

$$\left(\frac{G'}{G}\right)^2 : ad^2\alpha_1 - \alpha_1 - \frac{1}{2}c\alpha_1^2$$

$$\left(\frac{G'}{G}\right)^1 : ad^2\lambda\alpha_1 + bd\alpha_1 - c\alpha_0\alpha_1 - \lambda\alpha_1$$

$$\left(\frac{G'}{G}\right)^0 : -\alpha_1\mu + ad^2\mu\alpha_1 + bd\alpha_0 - \frac{1}{2}c\alpha_0^2$$

Solving this algebraic equation by Maple gives:

Case 1

$$c = c, \mu = \mu, \lambda = \lambda, \alpha_0 = 0, \alpha_1 = 0 \quad (9)$$

Case 2

$$c = \frac{2bd}{\alpha_0}, \mu = \mu, \lambda = \lambda, \alpha_0 = \alpha_0, \alpha_1 = 0 \quad (10)$$

Case 3

$$c = \frac{2(ad^2-1)}{\alpha_1}, \mu = \frac{\alpha_0\left(ad^2\alpha_0 - bd\alpha_1 - \alpha_0\right)}{\alpha_1^2\left(ad^2-1\right)}, \lambda = \frac{2ad^2\alpha_0 - bd\alpha_1 - 2\alpha_0}{\alpha_1\left(ad^2-1\right)}, \alpha_0 = \alpha_0, \alpha_1 = \alpha_1 \quad (11)$$

where $\alpha_0, \alpha_1, c, \lambda, \mu$ are arbitrary constants.

For case 2, substituting the solution set (Equation 10) and the corresponding solutions of Equation 4 into Equation 7 and also substitute $\xi = x - ct$ we have the solution of Equation 6 as follows:

When $\lambda^2 - 4\mu = 0$, we get the solution:

$$u(\xi) = -\frac{1}{2}\frac{\xi\lambda\alpha_1 - 2\xi\alpha_0 - 2\alpha_1}{\xi}$$

For case 3, substituting the solution set (Equation 11) and the corresponding solutions of Equation 4 into Equation 7 and also substitute $\xi = x - ct$ we have the solution of Equation 6 as follows:

When $\lambda^2 - 4\lambda\mu > 0$, we get the solution

$$u(\xi) = \frac{1}{2}\frac{\alpha_1\left(ad^2\sqrt{\frac{b^2d^2}{\left(ad^2-1\right)^2}}\cosh\left(\frac{1}{2}\sqrt{\frac{b^2d^2}{\left(ad^2-1\right)^2}}\xi\right) - \sqrt{\frac{b^2d^2}{\left(ad^2-1\right)^2}}\cosh\left(\frac{1}{2}\sqrt{\frac{b^2d^2}{\left(ad^2-1\right)^2}}\xi\right) + \sinh\left(\frac{1}{2}\sqrt{\frac{b^2d^2}{\left(ad^2-1\right)^2}}\xi\right)bd\right)}{(ad^2-1)\sinh\left(\frac{1}{2}\sqrt{\frac{b^2d^2}{\left(ad^2-1\right)^2}}\xi\right)}$$

and

$$u(\xi) = \frac{1}{2}\frac{\alpha_1\left(ad^2\sqrt{\frac{b^2d^2}{\left(ad^2-1\right)^2}}\sinh\left(\frac{1}{2}\sqrt{\frac{b^2d^2}{\left(ad^2-1\right)^2}}\xi\right) - \sqrt{\frac{b^2d^2}{\left(ad^2-1\right)^2}}\sinh\left(\frac{1}{2}\sqrt{\frac{b^2d^2}{\left(ad^2-1\right)^2}}\xi\right) + \cosh\left(\frac{1}{2}\sqrt{\frac{b^2d^2}{\left(ad^2-1\right)^2}}\xi\right)bd\right)}{(ad^2-1)\cosh\left(\frac{1}{2}\sqrt{\frac{b^2d^2}{\left(ad^2-1\right)^2}}\xi\right)}$$

When $\lambda^2 - 4\lambda\mu < 0$, we get the solution

$$u(\xi) = \frac{1}{2}\frac{\alpha_1\left(ad^2\sqrt{-\frac{b^2d^2}{\left(ad^2-1\right)^2}}\cos\left(\frac{1}{2}\sqrt{-\frac{b^2d^2}{\left(ad^2-1\right)^2}}\xi\right) - \sqrt{-\frac{b^2d^2}{\left(ad^2-1\right)^2}}\cos\left(\frac{1}{2}\sqrt{-\frac{b^2d^2}{\left(ad^2-1\right)^2}}\xi\right) + \sin\left(\frac{1}{2}\sqrt{-\frac{b^2d^2}{\left(ad^2-1\right)^2}}\xi\right)bd\right)}{(ad^2-1)\sin\left(\frac{1}{2}\sqrt{-\frac{b^2d^2}{\left(ad^2-1\right)^2}}\xi\right)}$$

and

$$u(\xi)=\frac{1}{2}\frac{\alpha_1\left(ad^2\sqrt{-\frac{b^2d^2}{\left(ad^2-1\right)^2}}\sin\left(\frac{1}{2}\sqrt{-\frac{b^2d^2}{\left(ad^2-1\right)^2}}\xi\right)-\sqrt{-\frac{b^2d^2}{\left(ad^2-1\right)^2}}\sin\left(\frac{1}{2}\sqrt{-\frac{b^2d^2}{\left(ad^2-1\right)^2}}\xi\right)+\cos\left(\frac{1}{2}\sqrt{-\frac{b^2d^2}{\left(ad^2-1\right)^2}}\xi\right)bd\right)}{(ad^2-1)\cos\left(\frac{1}{2}\sqrt{-\frac{b^2d^2}{\left(ad^2-1\right)^2}}\xi\right)}$$

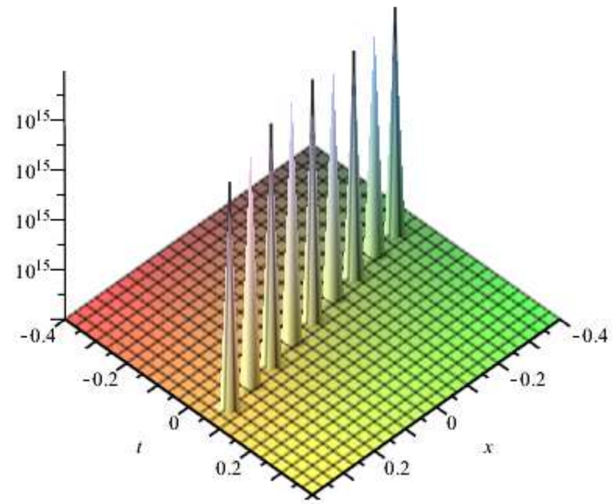

Figure 1. Graphical representation of the solution, when $\mu=1,\lambda=3,\alpha_0=1,\alpha_1=0,a=0,b=1,c=-.0001,d=.00005$ which is the heat equation.

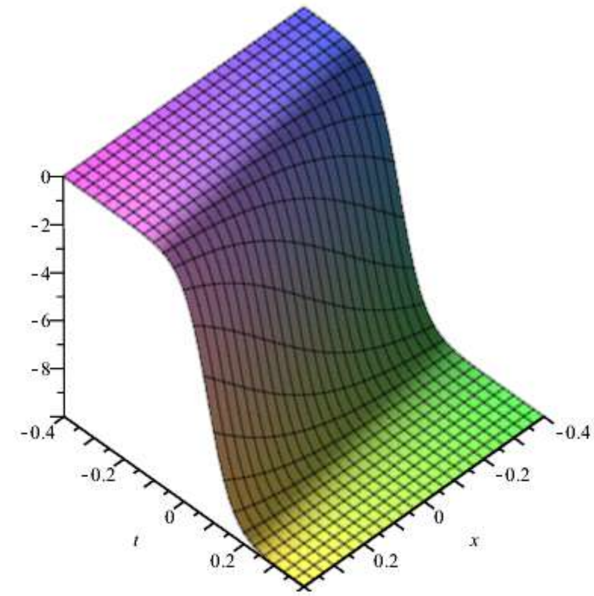

Figure 3. Graphical representation of the solution, which is the wave equation when $\mu=\frac{11}{4},\lambda=6,\alpha_0=1,\alpha_1=2,a=0,b=1,c=1,d=5$ ·

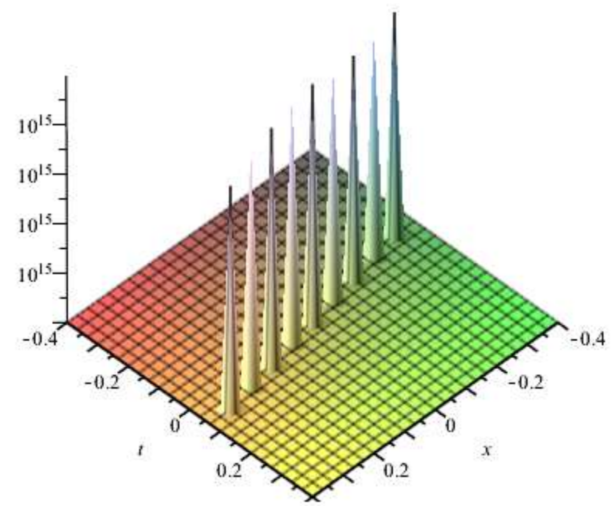

Figure 2. Graphical representation of the solution, when $\mu=\frac{11}{4},\lambda=6,\alpha_0=1,\alpha_1=2,a=0,b=1,c=1,d=5$, which is the heat equation and it is a periodic solution.

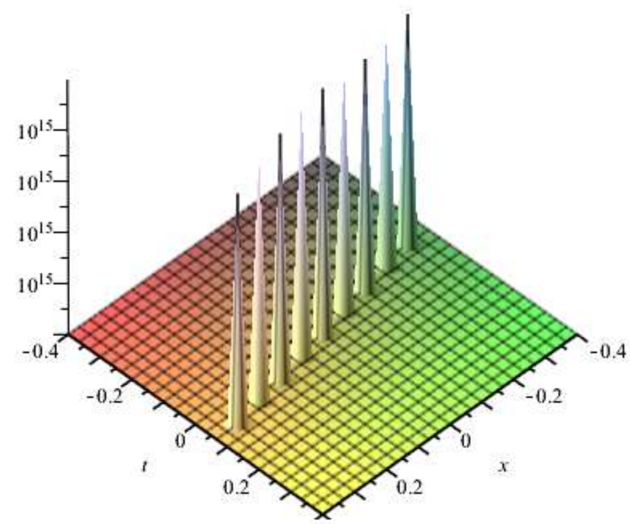

Figure 4. Graphical representation of the solution, which is the heat equation and it is a periodic solution, when $\mu=\frac{11}{4},\lambda=6,\alpha_0=1,\alpha_1=2,a=0,b=1,c=1,d=5$·

When $\lambda^2-4\lambda\mu<0$, we get the solution

$$u(\xi)=\frac{1}{2}\frac{\left(2ad^2-bd\xi-2\right)}{\left(ad^2-1\right)\xi}$$

And they are as shown in Figures 1 to 6.

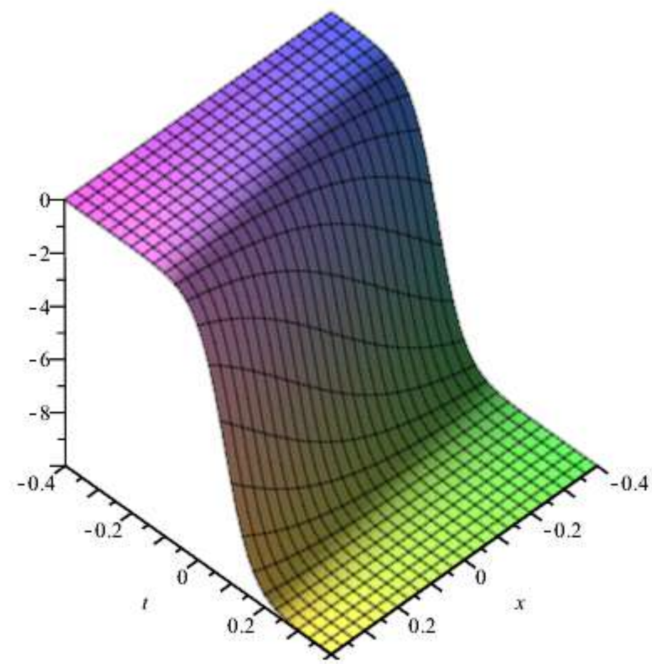

Figure 5. Graphical representation of the solution, which is the wave equation and it is a periodic solution, when $\mu=\frac{11}{4}, \lambda=6, \alpha_0=1, \alpha_1=2, a=0, b=1, c=1, d=5$.

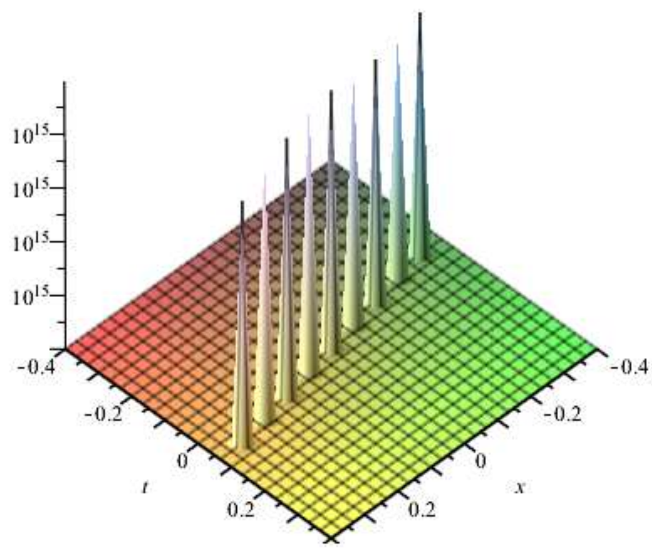

Figure 6. Graphical representation of the solution, which is the heat equation and it is a periodic solution, when $\mu=\frac{11}{4}, \lambda=6, \alpha_0=1, \alpha_1=2, a=0, b=1, c=1, d=5$.

Conclusion

In this paper, telegraph equations are investigated by using the generalized (G'/G)-Expansion method and various types of figures are shown such as wave, heat solutions of nonlinear evolution equations. Here, hyperbolic function, trigonometric function and rational function are also found. It is clearly said that, the generalized (G'/G)-Expansion method is more accurate in searching for exact solutions of nonlinear partial differential equations. We have come to the conclusion that (G'/G)-Expansion method is more convenient. In the similar way, we can represent graphically the behavior of other derived solutions where the exact solutions are solved with the help of software Maple.

CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

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